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This Journal is dedicated to the following aims:

1. Through published standard papers on the culture aspects, humanism and history of mathematics to deepen and to widen public interest in its values.
2. To supply an additional medium for the publication of expository mathematical articles.
3. To promote more scientific methods of teaching mathematics.
4. To publish and to distribute to groups most interested high-class papers of research quality representing all mathematical fields.

Noblesse Oblige

It is difficult to resist the conclusion that inherent in the nature of the mathematical process is something which furnishes an ever-ready pattern for analytic processes in widely varying non-mathematical fields. W. E. Byrne, writing to us recently, speaks of a series of articles that have been appearing in the *General Electric Review* on the use of "tensor analysis." In electrical science, he says, these articles speak of "dyadics, multiplication of matrices and the elements of tensors as Heaviside operators." Group Theory and other topics of interest to the pure mathematician are discussed.

Hermann Weyl writing to the late Emmy Noether, one of the great algebra minds of the century, is led to speak of Gordan's surprise at the "Analogy between binary invariants and the scheme of valence bonds in Chemistry." Weyl himself adds that, "Modern Quantum Mechanics recently has changed this analogy into a true theory disclosing the binary invariants as the mathematical tool for describing the valence states of a molecule in spin space."

There are possibilities for organized studies that search out those problems in every field whose solutions may be effected by the appropriate mathematics or mathematical method.

What we aim to make clear is that we are privileged as mathematicians to seek and discover the places and the times in the programs of a busy world where and when mathematics may *serve*.

It is questionable whether two chess players have a moral right to pursue their game while the neighbor's house burns down. If mathematical thinking is a pattern of true thinking, then may there not be a moral obligation resting on the mathematician to demonstrate this truth before the world so that the world may profit by its value, so that the vast potentials of mathematics may be applied to problems of crying human need as well as to those of science?

S. T. SANDERS.

On the Projection of an Angle Upon a Plane*

By F. A. RICKEY
Louisiana State University

Mathematical intuition is an almost essential tool of the mathematician. Yet, as with optical illusions, there are instances in which intuition alone may be misleading. Most of us have the feeling that the projection of an angle upon a given plane is not equal to the given angle unless the plane is parallel to that of the given angle. Nevertheless, it is true that any angle can be equal to its own projection upon a plane, provided only that the plane of the angle is not perpendicular to the given plane. We propose here to determine the magnitude of the projection of an angle under given conditions, to determine its maximum and minimum values, and the "position" the given angle must assume in its own plane in order that it shall be equal to its projection.

It will be sufficient to consider the case in which the plane of the projection passes through the vertex of the given angle, the projection on this plane being equal to that on all other planes parallel to it. Let α be the given angle and M the plane upon which it is to be projected. Let rectangular axes be set up in such fashion that the x-axis shall be the line of intersection of the two planes in question; with the vertex of α as the origin, the y-axis shall be taken in the plane of α; and finally the z-axis shall be so oriented, that if ϵ is the plane angle of the angle formed by the plane of α and by M, the unit circle in the yz-plane with center at the origin shall be intersected by the plane M at the points $(0, \cos\epsilon, -\sin\epsilon)$ and $(0, -\cos\epsilon, \sin\epsilon)$. Let ω denote the angle between the bisector of α and the positive x-axis. This angle will serve to define the "position" of α with respect to the line of intersection of the two planes.

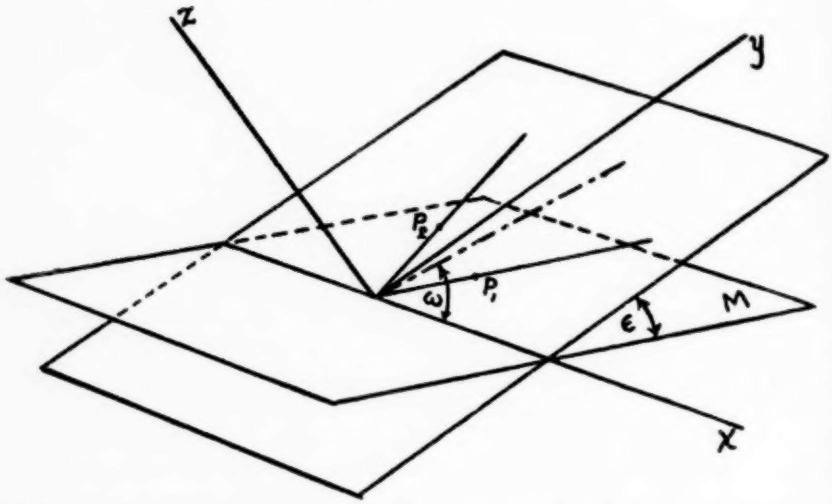
The direction cosines of the normal to M are evidently

$$0, \sin\epsilon, \text{ and } \cos\epsilon.$$

Hence the equation of M is

$$y\sin\epsilon + z\cos\epsilon = 0 \tag{1}$$

*The projection of an angle $P_1P_0P_2$ upon a plane M is defined to be the angle $P'_1P'_0P'_2$, where P'_1, P'_0, and P'_2 are the feet of perpendiculars from P_1, P_0, and P_2 respectively, to M.



If P_1 and P_2 are points on the respective sides of α at unit distance from the origin, their coordinates are,

$$(2)\qquad \begin{aligned} P_1 &: (\cos(\omega-\tfrac{1}{2}\alpha),\ \sin(\omega-\tfrac{1}{2}\alpha),\ 0)\\ P_2 &: (\cos(\omega+\tfrac{1}{2}\alpha),\ \sin(\omega+\tfrac{1}{2}\alpha),\ 0)\end{aligned}$$

Hence the parametric equations of the line through P_1 normal to M are

$$\begin{aligned} x&=\cos(\omega-\tfrac{1}{2}\alpha),\\ y&=\sin(\omega-\tfrac{1}{2}\alpha)+t\sin\epsilon\\ z&=t\cos\epsilon\end{aligned}$$

Solving (1) and (3) simultaneously, we obtain

$$t=-\sin\epsilon\sin(\omega-\tfrac{1}{2}\alpha),$$

which substituted in (3) yields the coordinates of P'_1, the projection of P_1 on M. The result can be written,

$$(4)\qquad P'_1 : (\cos(\omega-\tfrac{1}{2}\alpha),\ \cos^2\epsilon\sin(\omega-\tfrac{1}{2}\alpha),\ -\sin\epsilon\cos\epsilon\sin(\omega-\tfrac{1}{2}\alpha))$$

Similarly

$$(5)\qquad P'_2 : (\cos(\omega+\tfrac{1}{2}\alpha),\ \cos^2\epsilon\sin(\omega+\tfrac{1}{2}\alpha),\ -\sin\epsilon\cos\epsilon\sin(\omega+\tfrac{1}{2}\alpha))$$

From the well known formula

$$\cos P_1P_0P_2=\frac{h_1h_2+k_1k_2+l_1l_2}{\sqrt{h_1^2+k_1^2+l_1^2}\sqrt{h_2^2+k_2^2+l_2^2}}$$

where $h_1 = x_1 - x_0$, $k_1 = y_1 - y_0$, etc., the cosine of angle $P_1'OP_2'$ can be determined. This angle is the projection of α on M. Denoting it by α', we write after some simplification

$$(6)\qquad \cos\alpha' = \frac{1}{2}\cdot\frac{\cos\alpha(1+\cos^2\epsilon)+\cos 2\omega\sin^2\epsilon}{\sqrt{[1-\sin^2\epsilon\sin^2(\omega+\frac{1}{2}\alpha)][1-\sin^2\epsilon\sin^2(\omega-\frac{1}{2}\alpha)]}}$$

In order to determine maximum and minimum values of α' as a function of ω, it is more convenient to reduce (6) by means of various trigonometric identities to the form

$$(7)\cos\alpha' = \frac{\cos\alpha(1+\cos^2\epsilon)+\cos 2\omega\sin^2\epsilon}{\sqrt{(1+\cos^2\epsilon)^2+2(1+\cos^2\epsilon)\sin^2\epsilon\cos 2\omega\cos\alpha+\frac{1}{2}\sin^4\epsilon(\cos 4\omega+\cos 2\alpha)}}$$

Taking α and ϵ as constants, we find the derivative of $\cos\alpha'$ with respect to ω. The result can be simplified to,

$$\frac{d}{d\omega}(\cos\alpha') = \frac{-\frac{1}{4}\sin 2\omega\sin^2 2\epsilon\sin^2\alpha}{\left(\sqrt{[1-\sin^2\epsilon\sin^2(\omega+\frac{1}{2}\alpha)]\,[1-\sin^2\epsilon\sin^2(\omega-\frac{1}{2}\alpha)]}\right)^3}$$

Obviously this will be zero for $\omega = n\pi/2$. The change of sign from $+$ to $-$ as ω passes through the values $n\pi$ shows a maximum for $\cos\alpha'$ at $\omega = n\pi$. Similarly, the change from $-$ to $+$ at $\omega = \pi(2n+1)/2$ indicates a minimum for $\omega = \pi(2n+1)/2$.

Since

$$\frac{d\alpha'}{d\omega} = -\frac{d}{d\omega}(\cos\alpha')/\sin\alpha',$$

it follows that α' has a maximum value when $\cos\alpha'$ has a minimum, provided $\sin\alpha'$ is not zero for the corresponding values of ω. By setting $\cos\alpha' = \pm 1$ in (6), it can be shown that $\sin\alpha'$ can be zero when $\omega = n\pi/2$ only if $\alpha = 0$, $\alpha = \pi$, or $\epsilon = \pi/2$. We have thus shown that

$$\begin{aligned} &\alpha' = \text{max. for } \omega = \pi/2 \\ & \qquad\qquad\qquad (0<\alpha<\pi;\ 0\leqq\epsilon<\pi/2) \\ &\alpha' = \text{min. for } \omega = 0 \text{ or } \pi \end{aligned}$$

Substituting $\omega = \pi/2$ and $\omega = 0$ in (6), we obtain the respective results,

$$(9)\qquad \cos(\text{max. }\alpha') = \frac{1/2[\cos\alpha(1+\cos^2\epsilon)-\sin^2\epsilon]}{1-\sin^2\epsilon\cos^2\frac{1}{2}\alpha}$$

$$\text{(10)} \qquad \cos(\text{min. } \alpha') = \frac{1/2[\cos \alpha(1+\cos^2\epsilon)+\sin^2\epsilon]}{1-\sin^2\epsilon \sin^2\frac{1}{2}\alpha}$$

Adding the numerators and denominators of (9) and (10), we have the expression

$$\frac{\cos \alpha(1+\cos^2\epsilon)}{2-\sin^2\epsilon} = \cos \alpha.$$

From the fact that the value of $(a+c)/(b+d)$ is between that of a/b and of c/d, we see that the value of $\cos \alpha$ is between the values of $\cos(\text{max.}\alpha')$ and $\cos(\text{min.}\alpha')$. Since $\cos \alpha'$ is a continuous function of ω for $0 \leqq \epsilon < \pi/2$, it is necessary that $\cos \alpha'$ be equal to $\cos \alpha$ for some value of ω between 0 and $\pi/2$. To determine this value of ω, we set the expression for $\cos \alpha'$ given by (7) equal to $\cos \alpha$ and solve for ω. The result can be reduced to that of the following statement:

$$\text{(11)} \qquad \cos \alpha' = \cos \alpha \text{ when } \omega = \tfrac{1}{2}\cos^{-1}(-\cos \alpha \tan^2\epsilon/2)$$

Example. Let the projection of an angle of 60° be made upon a plane forming with the plane of the given angle a dihedral angle of measure 60°. The projection will be a maximum when the bisector of the given angle is perpendicular to the line of intersection of the two planes ($\omega = \pi/2$) and the magnitude of this projection is found by means of (9) to be $\cos^{-1}(-1/7) = 98°13'$ app. The magnitude of the projection will be minimum when the line of intersection of the planes bisects the given angle ($\omega = 0$) and its value from (10) is $\cos^{-1}(11/13) = 32°12'$ app. Again, the projection of this angle will be equal to 60°, according to (11), when ω, the angle formed by the bisector of the given angle and the line of intersection of the planes is $\frac{1}{2}\cos^{-1}(-1/8) = 49°48'$ app.

Corrigenda

The word "limit" in line 9, p. 6, Vol. XI of this magazine should read "unit".

The last two digits of N., i. e., 47, in line 2, p. 121, Vol. XI, should read 74.

The number "1837" in line 2, p. 206, Vol. XI, should read "1937".

The Sum of a Polynomial

By J. M. FELD
New York City

A finite sequence, $a_0, a_1, \ldots, a_n$, is said to be an arithmetic sequence of the pth order if its differences of the pth order are constant but not zero. The method of differences provides a formula for the sum of such sequences in terms of a_0, n, and $\Delta^k a_0$ $(k=1, 2, \ldots, p)$, $\Delta^k a_0$ representing the first of the kth differences. As is well known, the $(n+1)$th term of an arithmetic sequence of the pth order is a polynomial of the pth degree in n, $f(n)$. The problem of evaluating the sum of an arithmetic sequence is therefore equivalent to the problem of finding the value of $f(0)+f(1)+f(2)+\ldots+f(n)$. Stated in this manner it is simple, by means of nothing more complicated than Taylor's Theorem, to derive a formula for such sums in the form of a determinant. It is interesting to note that, since such sums include the sums of powers of the integers, the determinant obtained is, for special forms of $f(n)$, equivalent to Bernoulli's well-known formula for the sum of the pth powers of the integers. By the same method another formula for the sum of a polynomial in the form of a polynomial in $n+1$ will be derived.

Let $g(t)$ be a polynomial of degree $p+1$. Then by Taylor's Theorem

$$(1) \qquad g(t+1)-g(t)=g'(t)+\frac{1}{2!}g''(t)+\ldots+\frac{1}{(p+1)!}g^{(p+1)}(t).$$

Replacing t in (1) by the numbers $0, 1, 2, \ldots, n$ and summing the $n+1$ equations, we obtain

$$(2) \qquad g(n+1)-g(0)=\sum_0^n g'(t)+\frac{1}{2!}\sum_0^n g''(t)+\ldots\frac{1}{(p+1)!}\sum_0^n g^{(p+1)}(t).$$

If in (2) we substitute $f(t)$ for $g'(t)$, and note that

$$g(n+1)-g(0)=\int_0^{n+1} g'(t)dt,$$

equation (2) becomes

$$(2') \qquad \int_0^{n+1} f(t)dt=\sum_0^n f(t)+\frac{1}{2!}\sum_0^n f'(t)+\ldots+\frac{1}{(p+1)!}\sum_0^n f^{(p)}(t).$$

To simplify the notation, let

$$\sum_{t=0}^{n} f(t) = S_p(n) \quad \text{and} \quad \sum_{t=0}^{n} f^{(k)}(t) = S_{p-k}(n).$$

Then (2′) and the equations obtained by applying formula (2′) to $f^{(k)}(t)$, $k=1, 2, \ldots, p$, become

$$\begin{aligned}
&\int_0^{n+1} f(t)dt = S_p(n) + \frac{1}{2!}S_{p-1}(n) + \frac{1}{3!}S_{p-2}(n) + \ldots + \frac{1}{(p+1)!}S_0(n)\\
&\int_0^{n+1} f'(t)dt = \qquad\qquad S_{p-1}(n) + \frac{1}{2!}S_{p-2}(n) + \ldots + \frac{1}{p!}S_0(n)\\
(3)\quad &\int_0^{n+1} f''(t)dt = \qquad\qquad\qquad S_{p-2}(n) + \ldots + \frac{1}{(p-1)!}S_0(n)\\
&\quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot\\
&\int_0^{n+1} f^{(p)}(t)dt = \qquad\qquad\qquad\qquad\qquad S_0(n)
\end{aligned}$$

Regarding the $S_k^{(n)}$ as unknowns, we have a system of $p+1$ equations with $p+1$ unknowns. Cramer's formula gives us the value of $S_p(n)$ in the form of a determinant.

$$\sum_0^n f(t) = \begin{vmatrix}
\int_0^{n+1} f(t)dt & 1/2! & 1/3! & 1/4! & 1/(p+1)! \\
\int_0^{n+1} f'(t)dt & 1 & 1/2! & 1/3! & 1/p! \\
\int_0^{n+1} f''(t)dt & 0 & 1 & 1/2! & 1/(p-1)! \\
. & . & . & . \;\ldots & . \\
\int_0^{n+1} f^{(p)}(t)dt & 0 & 0 & 0 & 1
\end{vmatrix}$$

It is easy to evaluate this determinant inasmuch as its order can be repeatedly reduced by subtracting from the first column the last column multiplied by the element in the lower left-hand corner, and expanding by minors of the last row. However, an expanded form for $S_p(n)$, and therefore for the determinant as well, can be obtained

directly from (3). Let the numbers A_r $(r=1,2,3,\ldots)$ be defined as follows:

$$
(4)\qquad
\begin{aligned}
&1/2!+A_1=0\\
&1/3!+A_1/2!+A_2=0\\
&\cdot\qquad\cdot\qquad\cdot\qquad\cdot\\
&1/(r+1)!+A_1/r!+A_2/(r-1)!+\ldots+A_r=0\\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots
\end{aligned}
$$

By multiplying each equation in (3) successively by

$$1,\ A_1,\ A_2,\ldots,\ A_p,$$

respectively, and then adding, we obtain by virtue of (4)

$$S_p=\int_0^{n+1}f(t)dt+A_1\int_0^{n+1}f'(t)dt+A_2\int_0^{n+1}f''(t)dt+\ldots+A_p\int_0^{n+1}f^{(p)}(t)dt$$

It is natural to expect that the numbers A_r are closely related to the Bernoulli numbers, and this is in fact so. If, as is often done, we define the Bernoulli numbers by the symbolic equation $(1+B)^r-B_r=0$, $r>1$ where it is to be understood that in the expansion of $(1+B)^r$, B^k must be replaced by B_k, then the Bernoulli numbers satisfy the recurrent equations

$$
(5)\qquad
\begin{aligned}
&1+\binom{2}{1}B_1=0\\
&1+\binom{3}{1}B_1+\binom{3}{2}B_2=0\\
&\cdot\qquad\cdot\qquad\cdot\qquad\cdot\\
&1+\binom{r+1}{1}B_1+\binom{r+1}{2}B_2+\ldots+\binom{r+1}{r}B_r=0
\end{aligned}
$$

With the aid of the formula

$$\binom{n}{r}=\frac{n!}{r!(n-r)!}$$

the equations (5) may be replaced by

$$
\begin{aligned}
&1/2!+B_1=0\\
&1/3!+\frac{1}{2!}B_1+\frac{B_2}{2!}=0\\
&\cdot\qquad\cdot\qquad\cdot\qquad\cdot
\end{aligned}
$$

$$\frac{1}{(r+1)!}+\frac{1}{r!}B_1+\frac{1}{(r-1)!}\frac{B_2}{2!}+\frac{1}{(r-2)!}\frac{B_3}{3!}+\ldots+\frac{B_r}{r!}=0$$

By comparison with (4) it follows that $A_r=B_r/r!$. Since with this notation for the Bernoulli numbers, $B_{2n+1}=0$ for $n>0$, all the A_r with odd subscripts greater than unity vanish. We can now state for $p=2k$ and for $p=2k+1$

$$S_p(n)=\int_0^{n+1}f(t)dt+B_1\int_0^{n+1}f'(t)dt$$

$$+\frac{B_2}{2!}\int_0^{n+1}f''(t)dt+\ldots+\frac{B_{2k}}{(2k)!}\int_0^{n+1}f^{(2k)}(t)dt$$

or symbolically

$$S_p(n)=\int_0^{n+1}f(t+B)dt. \tag{6}$$

If $f(t)$ is not a polynomial but an analytic function, (6) still holds provided $f(t+B)$ is uniformly convergent.

It is easy to verify that (6) may be considered a symbolic expression of Euler and Maclaurin's summation formula. See, e. g., *The Calculus of Observations* by Whittaker and Robinson (1924) p. 134. When $f(t)$ is not a polynomial the Euler and Maclaurin formula, extended to an infinite number of terms, is not valid unless the remainder after n terms approaches zero with n, (loc. cit., p. 140). To insure this and to insure the validity of integrating the infinite series represented by $f(t+B)$, uniform convergence of this series is imposed.

Humanism and History of Mathematics

Edited by
G. WALDO DUNNINGTON

On Dieffenbach's Method for the Solution of Biquadratics

By WILHELM LOREY
Frankfort a. M., Germany

In 1821 the then Privatdozent of mathematics at the University of Giessen, Heinrich Wilhelm Dieffenbach, published a little sixteen page octavo memoir:

> *Anleitung zur allgemeinen Auflösung der biquadratischen Gleichungen nach Ferrari und Euler nebst einer neuen Auflösungsmethode.* Als Anhang zu den *Anfangsgründen der Algebra* von Fr. Wilh. Daniel Snell. Giessen bei C. G. Müller, 1821.

This memoir, an extract from an analytical thesis handed in to the faculty, on the basis of which Dieffenbach received the doctorate on February 26, 1820, in Giessen, is not mentioned in Ludwig Matthiessen's very exhaustive bibliography of algebraic writings: *Grundzüge der antiken und modernen Algebra der litteralen Gleichungen* (Leipzig, B. G. Teubner 1870; second, reduced-price edition, 1896). However it is cited by Christian Gottlob Kayser in his: *Vollständiges Bücherlexikon enthaltend alle von 1750 bis zum Ende des Jahres 1832 in Deutschland und den angrenzenden Ländern gedruckten Bücher.* (Leipzig 1834. Part II, p. 45). In the circles of mathematicians this memoir seems to be unknown. It may now be also very rare. However, the Giessen university library possesses a copy, on the basis of which Dieffenbach's "new process" shall be reported on here.

Dieffenbach assumes that the equation $x^4+Bx^2+Cx+D=0$, freed of the second term, can be factored thus:

$$(x^2+x\sqrt{p}+p-q)(x^2-x\sqrt{p}+p-t)=0$$

The auxiliaries result, by comparison with the coefficients of the given equation, from the equations

$$(1) \qquad B=-2p-q-t$$

$$(2)\qquad C=(t-q)^2\sqrt{p}$$

$$(3)\qquad D=p^2-pq-pt+tq$$

From (1) and (2) we get

$$t=-\frac{B+2p}{2}+\frac{C}{4\sqrt{p}},\quad q=-\frac{B+2p}{2}-\frac{C}{4\sqrt{p}}$$

These values substituted in (3) give for p the cubic:

$$p^3+Bp^2/2+(B^2-4D)p/16-C^2/64=0.$$

In the printed memoir there are two misprints in this cubic resolvent, the auxiliary equation, as Dieffenbach calls it: As a coefficient of $B\cdot p^2$ we find there 1/4 and for $-C^2$, 1/16. In the Giessen copy however these errors are corrected in handwriting, probably by Dieffenbach himself.

If p is one of the three "possible" roots (according to Dieffenbach's mode of expression), then by setting the quadratic factors equal to zero, we get the roots in the form

$$x_1=\sqrt{p}+\sqrt{q};\ x_2=\sqrt{p}-\sqrt{q};\ x_3=-\sqrt{p}+\sqrt{t};\ x_4=\sqrt{p}-\sqrt{t}$$

Of course, the ambiguity of the square roots in itself furnishes eight values, of which four are omitted. Strangely enough, Dieffenbach does not say so, while in the preceding description of Euler's process he makes the corresponding remark.

As an example Dieffenbach gives the equation (which Euler also has) $x^4-25x^2+60x-36=0$ with the cubic resolvent

$$p^3-\frac{25}{2}p^2=\frac{769}{16}p-\frac{3600}{64}=0.$$

This has the three real roots 9/4, 25/4, 4.

The first gives the roots of the biquadratic equation:

$$x_1=\sqrt{9/4}-\sqrt{1/4}=+1;\ x_2=\sqrt{9/4}+\sqrt{1/4}=2;$$

$$x_3=-\sqrt{9/4}-\sqrt{81/4}=-6;\ x_4=-\sqrt{9/4}+\sqrt{81/4}=+3.$$

The two other roots of the resolvent give, of course, the same values for x.

What is new in Dieffenbach's method? In Euler the roots of the biquadratic are represented by the sum of three square roots

$\sqrt{y_1}+\sqrt{y_2}+\sqrt{y_3}$ where y_1, y_2, y_3 themselves are the roots of the cubic resolvent. Dieffenbach forms a sum of two square roots, therefore even his cubic resolvent appears different from that of Euler. The factoring into two quadratic factors appears even in Descartes and his commentator Fr. v. Schooten. The latter makes the arrangement

$$x^4+Bx^2+Cx+D=(x^2+yx+z)(x^2-yx+v)=0$$

Hence the cubic resolvent becomes $y^6+By^4+(B^2-4D)y^2-C^2=0$. Between the v. Schooten and Dieffenbach auxiliaries there exist therefore the relations:

$$y=\sqrt{p}, \qquad z=p-q, \qquad v=p-t$$

The demonstrably very practical factoring of z and v is something new. Dieffenbach may have found it by proceeding from the assumed factoring of the roots of the biquadratic into the sum and difference of two square roots. If with the help of these the symmetric fundamental functions are set up, one obtains indeed the final equation and the above indicated relations between B, C, D, and p, q, t. Dieffenbach does not seem to have been acquainted with Descartes and v. Schooten. He mentions without accurate indication of sources, Ferrari and Euler, whose methods he describes very clearly. Cardan presented Ferrari's method in his *Artis magnae sive de regulis algebraicis* liber unus, Paviae 1545 Cap. XXXIX Reg. II. It consists of the transformation of the left side of the equation freed of the second term, into the sum of two squares. Euler first published his method in the memoir: *De formis radicum aequationis cuiusque ordinis conjectatio.* (Opera, Ser. I, Vol. VI, p. 4 seq.) It then passed over into his little *Lehrbuch der Algebra.* (Opera, Ser. I, Vol. I.)

Nothing further of a mathematical nature from the pen of Dieffenbach appeared, except a review of new mathematical tables, in the Frankfort *Didakalia* of 1823. He was born February 29, 1796, in Alsfeld and studied mathematics and finance in Giessen. He was active for two years in Teachers College at Friedberg. He later entered the department of tax administration and finally as a private gentleman indulged in traveling, having taken many long journeys, and carried on a versatile literary activity.

I recently became acquainted with the Dieffenbach memoir while I was writing a history of mathematics at Giessen in the nineteenth century for the *Nachrichten der Giessener Hochschulgesellschaft,* as a continuation of my memoir in that journal (Vol. 10, No. 2): *Aus der mathematischen Vergangenheit Giessens.* In this paper just mentioned the 17th and 18th centuries are treated, while the new memoir

is devoted to the period from 1800 up until the outbreak of the World War. My work *Der Briefwechsel von Leibniz mit Giessener Mathematikern,* also in the *Nachrichten der Giessener Hochschulgesellschaft,* Vol. 10, No. 3, serves as a supplement to the first memoir.

In the first memoir is information about Friedrich Wilhelm Snell, mentioned on the title page of Dieffenbach's book. Snell was a professor of philosophy, who wrote textbooks for "erste Anfänger." In J. G. Poggendorff's *biograpisch-literarisches Handwörterbuch für Mathematik, Astronomie, Physik, Chemie und verwandte Wissenschaftsgebiete* (Vol. 2, p. 949) this Snell is designated as a Giessen professor of mathematics, which is incorrect.

Notes on an 18th-Century English Mathematical Manuscript*

By A. W. RICHESON
University of Maryland

Among the manuscripts of the library of the University of Illinois is one purchased from Southern in 1918. This manuscript (43x29 cms.) consists of 291 folios, with graphs and diagrams, written in a beautiful 18th-century secretarial handwriting. The pages are bordered with red lines, and the numerous figures and diagrams are drawn with the utmost care and precision.

There is no title page, nor is there any indication of the identity of the author. The date of the manuscript is not given; however, from the astronomical problems, we should set the date at about 1756. On folio 4,r there is a beautiful pen drawing with the following notation: "*A Plan and Elevation of the Royal Academy in His Majesty's Dock Yard. Portsmouth.*"† The floor plan is below this drawing and at one corner in a contemporary handwriting "Date, about 1756. See Astronomical Examples." From this we should conclude that the manuscript was probably written by one of the instructors at the Royal Naval Academy at Portsmouth. The original manuscript was not foliated, nor was it divided into books or chapters. However, the subject matter lends itself to sectional division.

The first section on "*Arithmetick*", consisting of 60 folios, discusses the four fundamental operations of arithmetic with integers, denominate numbers, and fractions, both vulgar and decimal. At the beginning of the section the writer gives 21 definitions of the fundamental ideas of arithmetic and geometry, the first six of which are as follows:

1st. Whatever can be considered as capable of Augmentation or Diminution is call'd Quantity, under which May be Comprehended whatever can properly be said to have parts,

*The present writer wishes to take this opportunity to express his appreciation to the Director of the Library of the University of Illinois for his generous cooperation in allowing access to the above manuscript.

†What came to be the Royal Naval Academy at Portsmouth originated by an order of the Admiralty on March 13, 1729. The age of admission at this time was from 13 to 16 years. In 1806 it was reorganized as the Royal Naval College and in 1816 it was united with the School of Naval Architecture and continued until 1837 when the college was closed. In 1839 it was reopened for the training of officers, a function it retained till the college was transferred to Greenwich in 1872. (*Victoria History of English Counties*, London, 1912, Vol. 5, pp. 400-401; also Hammersly's *Naval Encyclopaedia*, Philadelphia, 1881, pp. 15-16.)

2nd. The mutual relation of two things of the same kind compared togather is call'd Ratio, and the similitude of Ratio is call'd Proportion,

3d. The knowledge of these comparisons of Quantity or the relation they have to one another is call'd Mathematics,

4th. All quantities have their parts either continuous or discrete, that is either United or separated, and that quantity which has it parts separated is call'd Multitude, and is the subject of a science call'd Arithmetick as that part which has its parts United is call'd Magnitude and is the subject of another call'd Geometry.

5th. If any quantity be considered as an indivisible it is call'd an Unit,

6th. A collection of Units is call'd a Number.

These are followed by definitions of such terms as aliquot part, multiple, aliquant prime number, composite prime, composed number (when one number will measure another, the latter number is a composed number), abstract number, constract number (a number that has a denomination), proposition, problem, theorem, axiom, postulate, corollary, lemma, etc.

Along with the four fundamental operations with numbers, the author discusses the four fundamental operations with denominate numbers. He gives numerous tables, followed by illustrative examples of the following type: "In 423 Miles, 5 Furlongs, 33 Poles, 14 Feet & 9 Inches, how many inches." Immediately following this, the process of finding square and cube root of integers is considered.

The discussion now turns to vulgar fractions. The definition of a fraction is stated as follows:

Any whole thing or Unit, may be considered as divided into any number of Equall parts, which have their Name from the number of them contained in that Unit....

Methods for the reduction of fractions are stated, then follow the four fundamental operations and the extraction of square and cube root.

From vulgar fractions the writer passes on to the "*Notation of Decimal Fractions,*" to which he gives the following definition:

All fractions that have their Denominators 10, 100, or 1000, i. e. Unity with any number of cyphers annexed are call'd Decimal Fractions, and their Denominators are never expressed but understood, and are separated from the whole Number by a Comma,* thus 54 3/10 is expressed 54,3 and 6 547/1000 = 6,547 and 65/1000 = ,065 when the numerator consists of as many figures as the Denominator has Cyphers.

*It should be noted that the use of the comma to mark off a decimal fraction was not common in England.

He then gives the rules for the four fundamental operations, extraction of roots, and applications of decimal fractions. Nowhere in this section does the text give a method to express a common fraction, whose denominator is other than some power of 10, as a decimal.

The author now discusses "*Ratio and Proportion*," giving a number of tables of aliquot parts and their application to exchange. At this point applications of the discussions and rules of the preceding sections are given; for example, a number of illustrative problems are taken from "Tare" and "Trett"*. A few pages further on the text turns to the calculation of interest. Definitions of both simple and compound interest are given along with the statements that compound interest was illegal and that 5% was the legal rate for simple interest in England at this time. It should be noted that compound interest tables are not used in any of the calculations and that only the practical methods are used in computing the compound amount for a fractional part of a conversion period and for computing the equated time for the payment of a single sum in lieu of several sums due at different dates. The section closes with a discussion of "Single and Double Fellowship," "Barter", and "Loss and Gain."

The second section is headed "*Logarithmical Arithmetick*" and consists of 4 folios. The writer gives a definition of logarithms along with rules for the characteristic, or index, as he calls it. These definitions and rules follow:

> If two [sic] a Bank of numbers in Arithmetick progression there be adopted a Bank of Numbers in Geometric progression, the former is call'd the Logarithm of the latter.
>
> That the Index of the Logarithm of any whole or mix'd Number, consists of as many Units as the left hand figures is distant from the place of Unity.
>
> But the Index of the Logarithm of a decimal Fraction, wants as many Units of an Hundred, as the left hand Significant is distant from the place of Unity.

Rules for the operations with logarithms, showing methods of interpolating the tables, and the use of logarithms in solving problems in proportion, are given. The writer does not use the negative characteristic as such, but $9-10$, or in many cases $99-100$; however the -10 or -100 is usually omitted.†

*"Tare" is the allowance for the container and "Trett" is the allowance for foreign particles in the weight of a commodity.

†Although the terms characteristic and mantissa are not used in the text, it should be noted that these terms were in general use before 1756, the approximate date of this manuscript.

The next section, consisting of 45 folios, is devoted to geometry. The author gives 37 carefully worded definitions, many of them illustrated by well-drawn figures, and then states four postulates as follows:

1. That from any point, to any other, a right line may be drawn,
2. That a finite right line, may be produced directly or continually,
3. That a Circle may be described on any Center, and at any Intervall,
4. If two quantities approximately so near to one another, that the difference between them is less than any assignable Quantity those Quantities are to looked upon as equal.

These postulates are followed by four axioms:

1. Those things which agree are equal.
2. All right angles are equal among themselves.
3. If upon two strait lines, a strait line falling, does make the internal angle on the same sides, less than two right Angles those strait lines being infinitely produced shall meet on that side where the Angles are less than two right ones.
4. Two right lines do not comprehend a space.

These postulates and axioms are followed by some work "on practical geometry," which consists principally of angle bisecting and construction problems. These problems range from the simplest types to the more complicated constructions of the ellipse, parabola, and hyperbola. The text then gives a number of theorems, without proof, on the straight line and triangle. These theorems are followed by a discussion of "Mensuration of Superficies" where several definitions of the units of length and area are given, after which the author goes into a discussion of the division of a given area into infinitely many parts.

The next section is taken up with trigonometry, which is defined as follows: "Trigonometry is that part of Geometry which teaches us to measure Sides and Angles of triangles." The law of sines and the law of tangents are stated without proof, and are followed by another well-known theorem on plane triangles and a rule attributed to Gunter.* The third theorem and Gunter's Rule are as follows:

3. In all Plane Triangles, if a perpendicular be left fall from any of its angles, on the opposite side of Base, if this perpendicular falls within, it will be, as *e/y* Base, is to the sum of the other two sides, as is the difference of those sides

*Edmund Gunter (1581-1626) was an English cleric who spent his leisure time in mathematical pursuits. In 1619 he was made Professor of Astronomy at Gresham College in London. (See *Braunmühl, Geschichte der Trigonometrie*, Leipzig, 1903, Zweiter Teil, p. 30.)

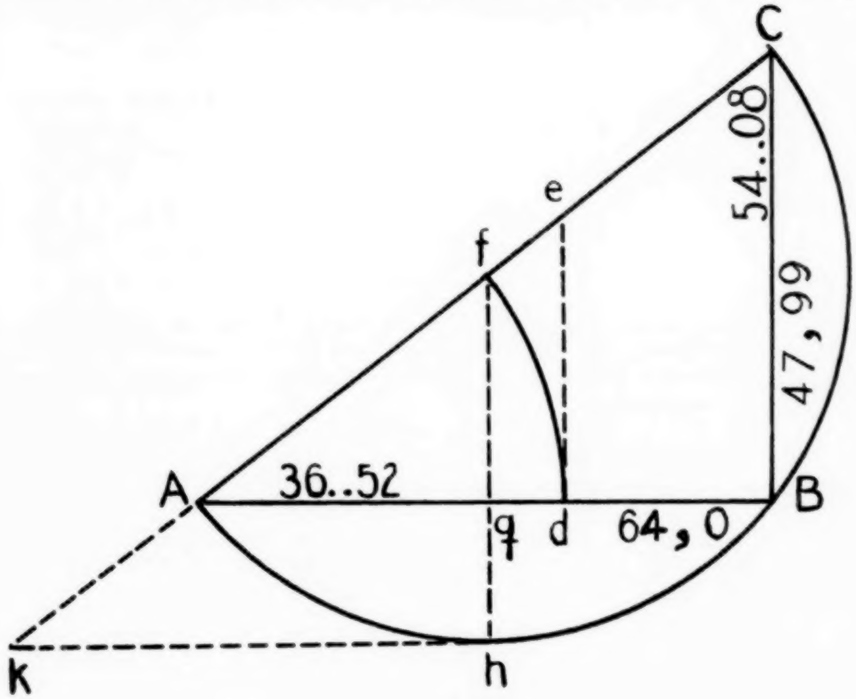
To the difference of the Segments of the Base,—But if the said perpendicular without the triangle it will be, as the Base, is to the sum of the other two Sides, so is the difference of the Sides, to the Alternate Base.....

To perform Theorem 1st: by Gunter. Say, as half the sum of the three sides, is to either of *e/y* containing Sides, so is the other containing side, to a fourth Number, the extent between this fourth Number & the difference, lay'd on the line of Versed Sines from 00 sheweth the required Angle

Or,

Say, as one of the containing sides, is to half the sum of the three sides so is the difference, to a fourth Number, so is Radius, to a Line against which on the line of Versed Sines, will be found the Angle sought.

The text then states that trigonometry may be divided into plane right-angled and plane obtused-angled trigonometry. Right-angled trigonometry is further divided into six cases with illustrative examples. Obtuse-angle trigonometry is also divided into six cases. The numerical computations are all performed by means of logarithms,



Fiq 1

and there is relatively little difference in the methods of solution for right triangles and oblique triangles. The following examples will illustrate the method of solution for the right triangle:

Case 1. Example. In the triangle *ABC*, right Angled at *B* are given, *AB* 64 Miles, the Angle *BAC* 36..52* Required *BC*.

1st & 2nd: triangles together...	*Ad* : *de* : : *AB* : *BC*
As Radius..10 &c............	10,000000
Is to tangt. *BAC*..36..52.....	9,875010
So is *AB*......64 Miles.......	1,806180
To *BC*......47,99 Do.........	1,681190

1st & 3d: triangles together....	*Ag* : *gf* : : *AB* : *BC*
As Sine of *ACB*....53..08....	9,903180†
To Sine *BAC*......36..52....	9,778118
So is *AB*............64......	1,806180
To *BC*.............47,99....	1,681190

1st & 4th triangles together....	*kh* : *hf* : : *AB* : *BC*
As tangt. *ACB*.....53..08....	10,126989
Is to Radius.......10 &c.....	10,000000
So is *AB*............64......	1,806180
To *BC*............47,99.....	1,681190

To these are all the proportions by which *BC* can be found, from what is here given, and these are in fact the same as would arise from making every side of the triangle Radius as it is commonly, tho very improperly express'd.

The following example will illustrate the method used for the solution of the problems in oblique triangles:

Case the Sixth. Three sides given, to find the Angles. .... In the triangle *ADE* are given *AE* 142, *AD* 104, and *DE* 70 Reqd. the angles *ADE*, *AED*, and *DAE*. By theorem: 3d.

*The notation 36..52 is used to denote 32°52′. Throughout the manuscript the degrees and minutes of an angle are represented in this manner. The present writer has been unable to find another instance where this notation is used.

†The log sine of angle $ACB = 53°8'$ is given as 9.903180; it should, of course, be 9.903108. However, this seems to have been corrected in the final result for the logarithm of *BC*, and probably is only a transposition of figures by the copyist.

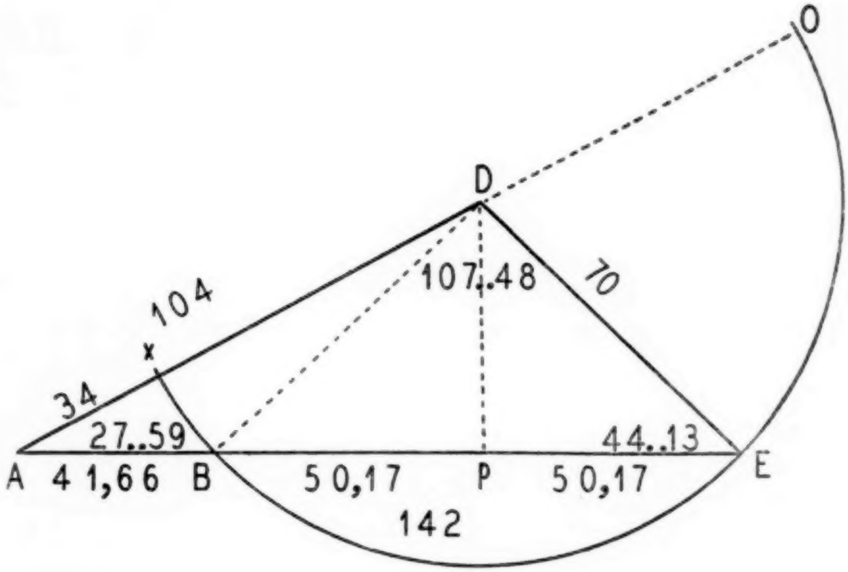


Fig 2*

As the Base *AE*....142.......	2,152288
Is the sum of other sides *AO*.....174...	2,240549
So is their difference *Ax*.......34......	1,531478†
To diff: of Segements.......41,66......	1,619939
Then in the Rt. angled triangled *ApE*	
It will be as *Ap*....91,83......	1,962984
Is to *AD*..........104.......	2,017033
So is Radius.......10 &c.....	10,000000
To Sect. *DAB*.....27..59....	10,054049

Again in the Rt. Angled triangle *DpE*	
It will be as *Ep*....50,17......	1,700444
Is to *ED*..........70........	1,845098
So is Radius.......10 &c.....	10,000000
To Sect. *DEp*......44..13	10,144654

*Fig. 2 is drawn as in the original manuscript. The numerical values of the parts of the figure are not clearly expressed; they are as follows: $AB=41.66$, $Bp=pE=50.17$, $AE=142$, $DE=70$, $Ax=34$, $AD=DO=104$, angles ADE, DEA, and DAE are 107°48′ 44°13′ and 27°59′ respectively.

†In this step log 34, which should be 1.531479 is given as 1.531478; furthermore, the addition or subtraction of the logarithms is incorrect, for log 41.66, which should be 1.619740 without interpolation is given as 1.619939.

AD	104	*AE* = 142	*Bp* = 50,17
DE	70	*AB* = 41,66	*AB* = 41,66
Sum sides	174	*BE* = 100,34	*Ap* = 91,83
	34	*Bp* = *pE* = 50,17	

Case the Sixth by Theorem Fourth*

As the product of the two contd: Sides *AD DE*†	4,169321
Is to ☐ of the ½ Z. of 3 Sides into difference	4,143139
So is the Radius Squared...............	10,000000
To Co-sine, Sq: of ½ the contd:..........	2)19,973818
Angle *DAE*................14..00	9,986909
2	
28..00	

A more expeditious way of performing Case the 6th: by the Rule followd: Theo. 4th:‡

Example, to find the Angle *DAE*.

To the Conpt: Arithm: of *AD*...104................	7,982967
Add Compt: Arithm: of *AE*.....142................	7,787721
& Log of ½ Z, 3 Sides...........158................	2,198657
& log *e/y* Difference............ 88................	1,973827
	2)19,973827
The Co-Sine of ½*DAE*....14..00..................	9,986913

The author now gives "*The Construction & Use of Gunters Scales.*"** It is stated as follows:

The principal line upon this scale is the line of Numbers, all the rest, vizt., those of Sines, Tangt., &c, are formed from it the line of Numbers is formed by taking the Logarithms of the Natural Numbers as they lye in order, from a Scale of Equall parts, & laying the off

*The manuscript is not quite clear here, since the only numbered theorems are from 1 to 3. The rule following Theorem 3 is headed as follows: "*To perform Theorem 1st*: by Gunter. ..." This is apparently the theorem referred to.

†Since the angle *DAE* is being computed, the sides referred to are *AD* and *AE*. It is also apparent that it is the product of these two sides and not the sum whose logarithm is 4.169321. In the next line the symbol in the shape of a parallelogram is used to indicate the product while "Z" (?) is used to denote the sum. This line would then read: "Is to the product of ½ the sum of the three sides into difference"; i. e., $s(s-DE)$ where $s=(AD+DE+AB)/2$. This method of computing the angle is nothing more than the well-known formula for the cosine of the half-angle.

‡Since there is no Theorem 4, the manuscript is not clear on this point. However, the method here is the cosine of the half-angle computed by means of cologarithms. In the third line the abbreviation "*Compt: Arithm:*" is for *Complementicum Arithmeticum*; i. e., arithmetic complement. It should be noted that the cologarithm of 142 is 7.847712 instead of 7.787721 and that the logarithm of 88 is 1.944483 instead of 1.973827. The sum of the correct logarithms will give 9.986909 the log cos 14.

**This scale is usually referred to as Gunter's "line of numbers." It was introduced about 1620 and was the precursor of the slide rule, described by Oughtred in 1632.

from any point assumed for the beginning of the line after this line is formed, the several points of the divisions of the Sines are found by looking out the Natural Sines of the several degrees and Minutes of the Quadrant on the line of Numbers and making a Mark on the line design'd to be the line of Sines & Thus the line of Sines are form'd: & in the same manner the Tangents &c may be found.

The use of these lines are to resolve in a more expeditious manner, every proposition where proportion is concern'd,—And the rule for resolving all sorts of proportions is by the Gunter, is to extend the feet of a pair of compasses from the first term of the proportion to the second & this extent will reach from the third to the fourth—or Answer.

The discussion now turns to spherical geometry which is given preparatory to the study of spherical trigonometry. In spherical trigonometry a number of definitions are stated, followed by 17 theorems. The author divides this topic into two parts; right-angled and oblique-angled spherical trigonometry with 16 cases under the right-angled and 12 cases under the oblique-angled.

The discussion continues with an application of trigonometry to "Problems in heights and distances." These problems are of the usual types used in determining the height or distance of an inaccessible object.

After a short topic on geography and the construction of maps and charts, 43 folios are devoted to navigation. The discussion is rather extensive and is illustrated with well-chosen examples and figures. It is followed by short discussions of astronomy, the Gregorian calendar, and the construction of dials. The problems in astronomy are rather simple. Problem 25 of the text will illustrate the type: "The Latd. of the place, day of the North & Altd. of the known Star given, to find the Azimuth & the time of Night." In the solution of this problem the year 1756 is used. The author then proceeds to the study of the tides. He gives a Mercator's sea chart and a careful pen drawing of a map showing the coast of Spain and Africa. This map is followed by some "Remarkable Observations and Accidents on board His Majesties Ship Britania Captn. F. R."*

Folios 264 to 376 are concerned with simple problems in elementary mechanics. The writer discusses the "Propositions of Motion", stating the fundamental laws of motion and their application to mechanical power. The laws for such simple machines as the lever, wheel and axle, pulley, screw, and wedge are given. These are followed by the composition of forces and a detailed consideration of the in-

*The Secretary of the British Admiralty informs the present writer as follows: "The Secretary of the Admiralty presents his compliments and in reply...., begs to state that in the 18th century ships of the Royal Navy named *Britannia* were in existance during the years 1682-1715; 1719-1749 and 1762-1812." This ccmmunicated is dated July 13, 1936.

clined plane. Numerous illustrative examples and drawings are given with each of the above.

The next topic considered is that of "Gunnery." The problems are simple, and for the most part concerned with the determination of the distance of a ship at sea from the coast or from some point inland. The method used is to count the number of pulse beats from the time a cannon is fired on the ship till the report is heard on the shore. After 10 folios devoted to this topic the manuscript closes.

This manuscript was probably prepared by one of the instructors at the Royal Naval Academy, while it was located at Portsmouth for use in his classes in mathematics. The text covers the elementary subjects of arithmetic, geometry, trigonometry, and applications to many allied topics. For the most part the text gives the theorems followed by wisely chosen illustrative examples, but generally leaves out the proofs entirely. In several instances the notation is unusual; for example, the comma to mark off a decimal fraction and two dots to indicate the degrees and minutes of angular measures. Usually the definitions and theorems are stated accurately and clearly, and are followed by appropriate examples and neatly drawn figures. On the whole the manuscript is written in a simple, clear-cut 18th-century style.

Announcement

We are pleased to announce that, beginning with our March issue, the Problem Department will be reinstalled. The successor to Prof. T. A. Bickerstaff, who resigned on account of his appointment to administrative work in the University of Mississippi, will be Prof. Robert C. Yates of the mathematical faculty of the University of Maryland, College Park, Maryland.

The selection of Professor Yates for the headship of this important Department of our Journal has been made after most careful deliberation, and we are confident that under his direction it will be maintained with scholarly vision, with judgment and effectiveness.

The Teachers' Department

Edited by
JOSEPH SEIDLIN

A Report on Present Tendencies in the Development of Mathematical Teaching in Japan, by M. Kuniyeda*

Review by JOSEPH SEIDLIN

The report is a fifty page booklet, 9″x6″, an epitomized draft of the following divisional reports:

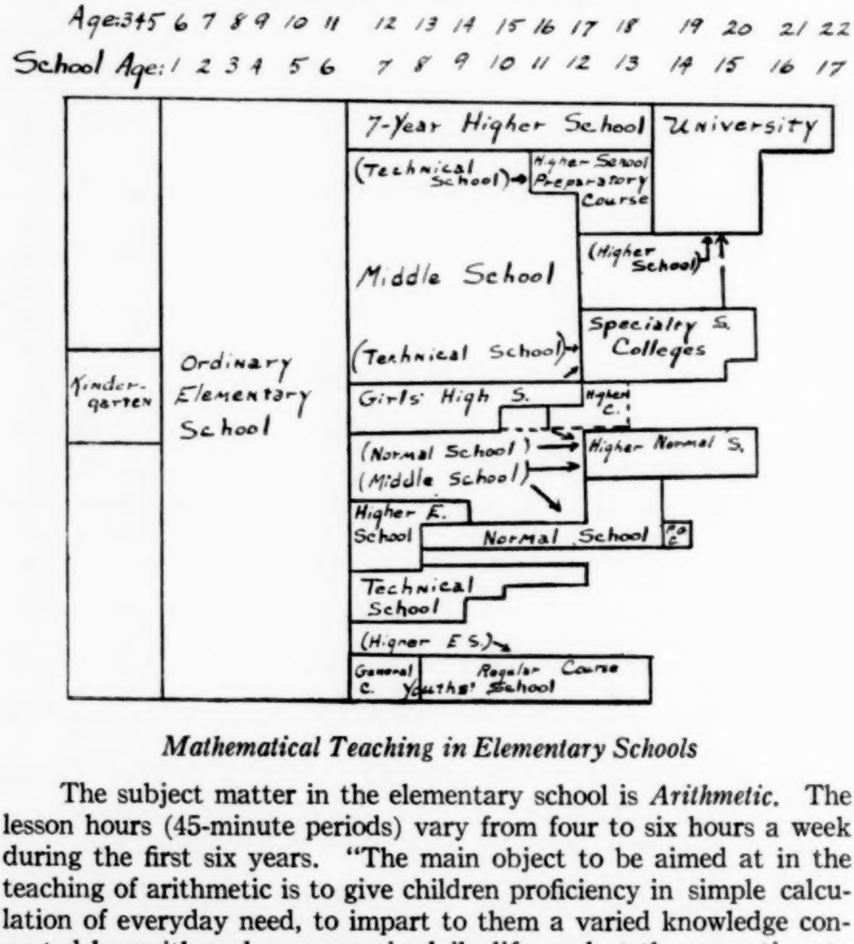
1. *Existing School System in Japan*, by I. Shimomura and K. Tsuda.
2. *Mathematical Teaching in Elementary Schools*, by N. Siono, T. Ando, and K. Tsuda.
3. *Mathematical Teaching in Middle Schools*, by M. Kuniyeda, M. Matsuo, N. Nabeshima.
4. *Mathematical Teaching in Girls' High Schools*, by R. Iwama and I. Nakazawa.
5. *Mathematical Teaching in Technical Schools*, by M. Watanabe and K. Mizu'uti.
6. *Mathematical Teaching in Youths' Schools*, by Miss K. Horiguchi.
7. *Mathematical Teaching in Higher Schools*, by H. Watanabe.
8. *Mathematical Teaching in Colleges and Specialty Schools*, by M. Watanabe.
9. *Mathematical Teaching in Universities*, by K. Sugimura.
10. *Mathematical Teaching in Normal Schools*, by Yayotaro Abe.†
11. *Training of Mathematical Teachers*, by N. Nabeshima.
12. *Societies, Associations, and Publications Concerning Mathematics*, by N. Nabeshima.

"In order to facilitate the reader's understanding of the conditions of mathematical teaching in various sorts of schools in Japan, a table of the Japanese School System will be given."

*Dr. Motoji Kuniyeda, Professor in the Tokio University of Literature and Science is the chairman of the Japanese National Commission on the Teaching of Mathematics.

†Professor Abe wrote the article on primary and secondary education in Japan for the Fourth Year Book of the National Council of Teachers of Mathemazics, Bureau of Publications, 1929.

Mathematical Teaching in Japan
A Table of the Japanese School System



Mathematical Teaching in Elementary Schools

The subject matter in the elementary school is *Arithmetic.* The lesson hours (45-minute periods) vary from four to six hours a week during the first six years. "The main object to be aimed at in the teaching of arithmetic is to give children proficiency in simple calculation of everyday need, to impart to them a varied knowledge connected herewith and necessary in daily life, and at the same time to habituate them to be clear and exact in thinking."

How is all this to be achieved? "In order to accomplish this object, a School Book Bureau is established in the Education Ministry, which is charged with the compilation of State text-books for the mathematical teaching. . . ."

Strange as it may seem to us, the leitmotif of the tendencies and trends in modernizing mathematical teaching in Japan is given in the preceding paragraph. Our own educators often resent the implicit authority of the text-book. There are those who claim, lament and

decry, the relative impotence of courses of study, syllabi, aims and objectives, researches and published findings in the improvement of teaching, pitted against the dictatorial authority of the text-book. In Japan, however, the text-book is avowedly the authority, the pride, and the hope for improved teaching. We must keep in mind, however, that in Japan the text-book is a national institution. Evidently, a text-book is a legitimate document only when it is fathered by the Education Ministry. If the teachers of mathematics in Japan are as afflicted by text-book-phobia as their brethren in this country, their enforced resistance to the disease needs must evoke our sympathy.

On page 11 of the report we find, accordingly, this statement: "In view of the recent tendencies... of educational development, a substantial revision of arithmetical text-books has been attempted by the authroities of the Education Ministry in the hope to keep these books in line with the general developments made in the national life." It is quite desirable therefore to reproduce, in part, at least, the nature of the reported revision.

1. "The main object in view is to cultivate children's thinking on the basis of mathematical principles.
3. The contents and their treatment should be kept in line with practical life.
8. Text-books should be made more appellant to the interests of children.

   As compared with the earlier ones, the revised text-books of arithmetic have undergone substantial changes in many points. Increasing signs of enthusiam are being manifested in the educational world of the country in welcome of the new text-books, which marks a bold leap taken in the development of mathematical teaching in Japan."

Mathematical Teaching in Middle Schools

In the section on mathematical teaching in the middle schools, much of the substance and some of the language of the "General Principles and Recommendations" of our National Committee on Mathematical Requirements‡ is in evidence.

The revised outlined program provides the usual topics. It is of interest to us, however, to note the "salient features"

1. The appreciation of synthesis.
   ...It is stated in the precautionary (?) notes attached to the new program that the idea of function should be made the center of synthetic practices.

‡The Reorganization of Mathematics in Secondary Education; A Report by the National Committee on Mathematical Requirements, under the auspices of the Mathematical Association of America. 1923.

2. Only major items have been shown in the program.
   ... With details of the teaching program left for compilation to the discretion of instructors and editors of text-books...
3. Practicability has been stressed.
   (a) Instruction in algebra is made to center around equations and most of those difficult problems relating to mere formalities have been deleted.
   (b) The introduction of numerical trigonometry has been expedited.
   (c) Special regard has been paid to the fostering of functional ideas in children resulting in considerable increase in graphic materials.
4. Attention has been brought to the degrees of development of pupils' capacities. A striking indication hereof will be seen in the insertion of Geometrical Figures in the geometrical stuff (topics ?) in the new program, which is chiefly to be dealt with through intuitional practices.

It is in this section of the report that we find a direct and candid admission that all has not been well with the teaching of mathematics in Japan: "It has often been said that enthusiasm for studies in teaching methods is lacking among those who are engaged in mathematical instruction in the country." However, certain influences are at work to improve the situation. In particular, the following two "tendencies" are "summarized."

1. Study of the manner in which teaching materials should be presented before the pupils.

Without indulging in traditional practices in mere deduction and inference by giving the students to understand the stuff in text-books after interpretations and explanations given by teachers and bringing them to attempt a solution of problems according to fixed formulas, plans are under way to demonstrate before the scholars retroactively the processes in which the nature of the materials and the relations existing among them have been composed in the presented forms, thus facilitating the learners to retrace on their part the said processes. In other words, pupils should be initiated in the nature of teaching materials and their relations through the exercise of intuition or experiments and actual measurements as occasion calls, so that they should be able to present the results of their studies in systematical forms and led to discovering theorems and laws....

2. Progress made in the method of considering the proficiency of students.

It was in 1923 that the Mathematical Association of Japan for Secondary Education appointed a special committee to investigate into the disputed objectivity of the traditional system of marking students' proficiency by the medium of examination papers. ... increasing attention has since been paid throughout the country to the

possibilities of improving the methods of examination. It may be regarded as a marked improvement made in this direction that the "diagnostic test" has been brought into consideration with a view to extending beneficiary instruction to repair the injustices, if any, done to the pupils under the old system. . . .

Mathematical Teaching in Girls' High Schools

"Higher ordinary education for females in Japan has been left considerably behind that for males. An indication may be seen in the fact that the ratio between males and females who are given higher ordinary education in the country was put in 1917 at 100:54. Female education has, however, since made rapid progress until at last in 1931 the ratio was given as 110:100 in favor of the females."

It seems, furthermore, that equality of opportunity in the education of both sexes has been achieved in mathematics only. Or is it that mathematics is not made especially suitable to "acquisition of matters necessary for domestic life"? Be that as it may, we are confronted with a "notable feature": ". . ., despite the distinct standards existing between the middle schools and the girls' high schools in the country in other courses of secondary education, the latter chiefly being aimed at the acquisition of matters necessary for domestic life, instruction in mathematics is leading the male and the female towards an indiscriminate field of education."

Mathematical Teaching in Technical Schools

"Technical schools of secondary grade in Japan are divided into Agricultural, Technological, Commercial, Navigation, Fisheries, and other schools. Agricultural schools are subdivided into several courses, while technological schools comprise more than ten different divisions. The course of studies in these schools ranges from three to five years. *No fixed program of instruction concerning these schools has yet been given by the Education Ministry, with each school left to devise its own plan according to practical requirements.*"

In general, the content of the mathematical courses in the technological schools is practically the same as in the middle schools. In the fifth year courses in analytic geometry, the differential and integral calculus are included. The slide rule, the *Soroban,* and other calculating machines are employed. In particular, courses in mathematics are coordinated with technological courses. In recent years there has been a steady increase in the enrollment in the technical schools. Indirectly this has been responsible for increased activity and more rapid progress in the teaching of mathematics.

Mathematical Teaching in Youths' Schools

These schools only recently established are still in their period of "formation". One of the objectives, that of developing or improving ordinary (?) knowledge, skills, and culture, makes provision for mathematical instruction. Some sort of course in general mathematics beyond the bare necessities for daily use, is indicated. As yet no satisfactory text-books to meet this need have been published and there is no regulated fixed program.

Mathematical Teaching in Higher Schools

"The object of mathematical teaching in the Higher School is to furnish the students with such common sense and culture as would make of them capable members of the community provided with sufficient ability to understand adequately present-day culture and bear themselves properly with the national situation, and at the same time to give them such basic knowledge as would enable them to acquire special knowledge in the advanced courses in universities."

We wish our colleagues in Japan great and unqualified success in the attainment of these objectives. At any rate we congratulate them on their courage, intrepidity, and hardihood which so open pronouncement of such ideal and ambitious aims needs must have required.

Perhaps because of such exacting and ideal objectives things have not gone as well as expected or hoped for. The Higher School, therefore, receives some harsh criticism.

"In spite of the professed object of completing the higher general education, Higher Schools in this country are developing tendencies to embody, as a matter of fact, preparatory schools for universities, and, as compared with the elementary and secondary educations, considerable inertia prevails in the enquiries into the educational system of Higher Schools."

It seems to me our own Liberal Arts colleges have been accused in similar and more or less so bold fashion. Evidently, our Japanese colleagues are more determined than we have been to "remedy" the situation.

"Acting on instructions from the Education Ministry" (We do not have any such) "several professors of the universities well-versed in mathematical education" (We have such) "have recently inspected actual conditions of mathematical teaching in Higher Schools throughout the country. In compliance with the recommendations advanced by these professors upon completion of their inspections, a short course

on the methods of instructing in mathematics in the Literature Course of the Higher School was held in the capital, to which were called most of the mathematical instructors from Higher Schools in the country. With this as the first step, the authorities of the Education Ministry are expected to embark hereafter upon an active reform program for the benefit of the mathematical teaching in the higher general education." American journals of mathematics, please copy!

Mathematical Teaching in Colleges and Specialty Schools

"One of the outstanding features of the actual instruction in mathematics being conducted in these schools in recent years is the importance attached to the mathematical course as providing a common basis for all other special studies in these schools, while on the other hand signs of so many different branches of mathematics being fused into a whole have also been increasingly manifest together with the popularization of the higher mathematics and the cultivation of practicability of the ordinary mathematics."

Some of the "latest developments" in mathematical teaching in these schools are:

1. The introduction of mathematics in courses which had formerly employed little or no mathematics.
2. The whole outline of the calculus is taught in the junior classes.
3. Partial differentiation, differential equations, and Fourier Series are regularly taught subjects.
4. Graphical calculation, nomography, statistical mathematics, and the method of least squares have also been introduced. Some of the specialty schools are provided with mathematical laboratories.
5. . . . most of these schools manage to drive home general principles of mathematics with the select few materials, with a good number of hours being allotted to problems.

Mathematical Teaching in Universities

In 1912 only three universities in Japan, the Imperial Universities at Tokyo, Kyoto, and Tohoku, provided graduate instruction in mathematics. At present, however, there are seven universities which offer graduate work in mathematics. Also, the number of students completing these mathematical courses total 80 to 90 annually which is 7 to 8 times as many as 25 years ago.

Before graduating at each university (In addition to the three mentioned above, they are: the Hokkaido, the Osaka, and the Tokyo and Hiroshima Universities of Literature and Science, called Bunrika Daigaku) a student is required to have been three years in residence

as well as to pass examinations in the compulsory and a certain number of optional subjects. (See the accompanying table, Mathematical Course in the Tokyo Imperial University.)

Mathematical Course in the Tokyo Imperial University

COMPULSORY SUBJECTS	*Fixed Period of Study (Year)*	*Lesson Hours Per Week*	*Exercise Number of Sitting Per Week*
Differential and integral calculus	1	4	1
Higher algebra	1	2	1
Higher geometry	1	3	1
Theory of functions	1	2	1
Theory of differential equations	1	3	1
Dynamics (Part I)	1	2	1
Special lectures on mathematics	1	2	
Mathematical seminar	1		
Optional Subjects. Students are required to finish more than two of the following subjects:			
Higher algebra and number theory	1	3	1
Synthetic geometry and descriptive geometry	1	3	1
Theory of probability and statistics	1	2	1
Spherical astronomy and the method of least squares	1	3	
Celestial mechanics	1	3	
Dynamics (Part II)	1	2	
General physics	1	3	
Experiments in physics	½		

Mathematical Teaching in Normal Schools

"The object of these normal schools which have been established and are maintained by Prefectural Governments in Japan is to train teachers for elementary schools." Its course of study extends over five years for graduates of the higher elementary school of the two years' course; it extends over two years for graduates of middle schools and girls' high schools. The objectives of mathematical teaching, as given in the revised regulations of 1931, are like those in the secondary schools.

The program for the two-year course is given as follows:

First Year (2 hours a week for males, 3 hours a week for females)

Progressions, logarithms, solid figures, trigonometrical functions, daily computations relating to percentage, methods of teaching arithmetic in elementary schools.

Second Year (2 hours per week)

Synthesis of the teaching materials already taught and supplements to them, inequalities, maxima and minima, conic sections, ellipsoid, and study of arithmetical teaching materials for elementary schools.

Higher Normal Schools

"The object of Higher Normal Schools is to train teachers *of intermediate schools*, viz., of normal schools, middle schools, and girls' high schools. There are two schools of this kind, one in Tokyo and the other in Hiroshima."

"The following table outlines the mathematical curriculum... of the Science Department of the Tokyo Higher Normal School."

The Mathematical Course of the Science Department of the Tokyo Higher Normal School

School Year I	*II*	*III*	*IV*
Arithmetic (2)........	Algebra (2)........	Algebra (3)........	Differential and Integral calculus (2)
Algebra (2)...........	Geometry (2)......	Geometry (3)......	Advanced Calculus (4)
Geometry(3)..........	Analytical geometry (3)	Differential and Integral calculus (4)	Study of Mathematical teaching (3)
Trigonometry (2)......	Differential and Integral Calculus (4)	Exercise in Algebra (2)	Applied Mathematics (2)
Analytical geometry (2)	Exercise in Algebra (2)	Exercise in Geometry (2)	Exercise in Mathematics (2)
Total hours per week 11	13	14	13

Training of Mathematical Teachers

(1) Mathematical Instructors of Higher-Grade Schools.

"Those who complete the mathematical courses of the Bunrika Daigaku or of the Science Faculties of the Tokyo, the Kyoto, the Tohoku, the Hokkaido and the Osaka Imperial Universities, are given qualifications without examination to instruct in mathematics in Higher Schools, Colleges and Specialty Schools. The examination for the liecnse of higher-school mathematical teachers is held by the Education Ministry once in a few years. Three men passed" (We are not told how many failed) "the license examination of this kind in 1935."

(2) Mathematical Teachers of Intermediate Schools.

The examination for the license of mathematical teachers in the intermediate schools is held once a year by the Education Ministry. At present the materials for the examination are taken from arithmetic, algebra, geometry, trigonometry, analytical geometry, and the differential and integral calculus. The part of the examination testing on teaching methods is oral. The number of successful examinees (again we are not told the number of unsuccessful examinees) in 1935 is put at 20.

The license is granted without examination to graduates of the Tokyo Physics School (a private institution) as well as to those who studied more than the fixed number of subjects on mathematics in the Faculties of Science, Technology and Agriculture of the Imperial University, and also to those who have concluded the study of a sufficient number of mathematical subjects in the technical universities and certain technical colleges.

"With a view to improving *the scholarship* of teachers the Education Ministry holds short courses taking advantage of summer vacation. The subjects of the summer school held in 1935 included the number concept, the general idea of synthetic geometry, general theory of the integral equations, and various problems on modern mathematical education. Those who attended last summer school on mathematics numbered 300, which represents a striking proportion" (striking in what way?) "to the whole number of existing qualified teachers of mathematics in the country which is estimated at about 3,000."

Conclusion

"In conclusion, it may be said that despite the fact that Japan seemed to have been left some time in the past about 20 years behind the European and American nations in starting the movement for reforming mathematical teaching, she has made steady progress in this direction since 1918 until at last at the present time Japan may take pride in being devoted to assiduous studies on mathematical teaching, keeping her position on the foremost front of the mathematical education in the world and yet without being affected by the reactionary thought prevailing in various parts of the world." We make no comment.

Mathematical World News

Edited by
L. J. ADAMS

The monthly pamphlet, *Pitagoras*, began its second volume with the September, 1936 number, which was devoted to an expository treatment of the division algorithm and a discussion of the Gaussian curve. This interesting periodical is published and edited by Jorge Quijano at Liverpool, 30, Ciudad de México, México.

The American Mathematical Society announces meetings as follows:

1. New York City, February 20, 1937. Professor T. R. Hollcroft will address the meeting on the subject *The existence of algebraic plane curves with given singularities.*
2. New York City, March 26-27, 1937.
3. Stanford, California, April 3, 1937.
4. Chicago, Illinois, April 9-10, 1937.

Professor W. T. Reid will read a paper on *Boundary-value problems in the calculus of variations.*

The fall meeting of the Maryland-District of Columbia-Virginia Section of the Mathematical Association of America was held at the National Bureau of Standards, Washington, D. C. on Saturday, December 5, 1936.

The following six papers were read:

1. *Hyperconformal transformations.* G. F. Alrich, University of Maryland, introduced by the Secretary.
2. *An elementary transformation of rectangular axes.* Professor J. W. Blincoe, University of Virginia. (Read by title only, in the absence of Professor Blincoe.)
3. *On the role of a basis in vector analysis.* Professor J. H. Taylor, George Washington University.
4. *Certain factorial sums.* Dr. S. Kullback, Office of the Chief Signal Officer, Washington, D. C., introduced by the Secretary.
5. *An episode in the life of Sylvester.* Professor R. C. Yates, University of Maryland, introduced by the Secretary.
6. *Cremona transformations and their applications to algebraic function theory.* Professor A. E. Landry, Catholic University of America.

News items from The University of New Mexico include the following:

1. Dr. H. D. Larsen of the University of Wisconsin has recently been appointed head of the division of statistics within the department.

2. In October, the University of New Mexico Press published a bulletin entitled, *An Introduction to Mathematics*, and written by Dr. C. V. Newsom, professor of mathematics. This bulletin gives a popular exposition of the nature of mathematical knowledge.

3. Professor Newsom has recently been appointed as a visiting lecturer to about twelve institutions in the southwest as representative of the newly organized Southwestern Section of the Mathematical Association of America. Dr. Newsom expects to make his trip in April, 1937 and he will lecture upon some aspect of mathematical philosophy.

The following addresses were delivered before the October 23, 1936 meeting of the Mathematics Club of the University of Wisconsin.

1. *The Problem of Bolza in the Calculus of Variations.* Professor G. A. Bliss, University of Chicago.

2. *Some Geometrical Features of the Calculus of Variations.* Professor C. Carathéodory (University of Munich).

Dr. T. Vijayaraghavan, Dacca University, India, who is touring the United States under the auspices of the American Mathematical Society, lectured November 19-20 at the University of Illinois on *The Ancient Civilization of Southern India* and *The Rate of Increase of Real Solutions of Differential Equations.*

Johannes Franz Josef Tropfke, noted mathematical historian of Berlin, celebrated his seventieth birthday on October 14. He received his doctorate at the University of Halle in 1889. Volume III of the third edition of his history of elementary mathematics is expected to appear soon.

Professor Robert E. Moritz of the University of Washington has been commissioned by the Educators' Association Incorporated to prepare an article on trigonometry for the forthcoming (January 1, 1937) edition of the Volume Library which is an encyclopedia for grammar and high school students. Professor Moritz, it will be recalled, is the author of a very comprehensive trigonometry published by Wiley, *Memorabilia Mathematica* published by Macmillan, and in addition is the inventor of the cycloharmonograph as well as a con-

tributor of numreous research articles in the technical journals. In the preparation of the article which will be more than usually complete (5,000 words), Professor Moritz will be assisted by Dr. Hermance Mullemeister, assistant professor of mathematics at the University of Washington.

The annual index of *The American Mathematical Monthly* for volume XLIII is included in the December, 1936 number.

The annual meeting of the Philadelphia Section of the Mathematical Association of America was held in Room 201, Bennett Hall, University of Pennsylvania, Saturday, November 28th, at 10:30.

1. *Remarks on Abstract Spaces.* Dr. J. A. Clarkson, Pennsylvania. (Introduced by Professor Oakley).

2. *Triangulation and Related Problems.* Professor S. S. Cairns, Lehigh.

3. *Fiducial Argument in Statistical Inference.* Professor S. S. Wilks, Princeton. (Introduced by Professor Oakley).

4. *The Undergraduate Comprehensive Examination.* Professor W. Rue Murray, Franklin and Marshall. (Introduced by Professor Long).

Professor William H. Roever sends the following news items from Washington University, Saint Louis:

1. Dr. F. A. Butter, formerly at Stanford University, California, is spending the academic year, 1936-37, as Research Assistant to Prof. Szegö at Washington University, St. Louis, Mo.

2. The Doctor's degree was conferred on Joseph Selwyn Rosen, in June, 1936. Mr. Rosen received his B. S. and M. S. at Washington University. The title of his thesis is: *Some Generalizations of Bessel Functions.* His major subject was mathematics, minor, physics.

3. The Department of Mathematics at Washington University is offering the following advanced courses in the Summer Session of 1937:

Analysis (of the type of Volume I of Goursat-Hedrick's Analysis).

Projective Geometry (of the type of Winger's Projective Geometry).

Advanced Descriptive Geometry (including the mathematical theory of making pictures of space objects).

Advanced Differential Equations (of the type of Volume II, Part II of Goursat-Hedrick's Analysis).

Classes in the Summer Session begin June 21 and continue six weeks.

4. Two lectures were given at Washington University by Dr. T. Vijayaraghavan, of the University at Dacca, Bengal, India: *The Mathematics of India* and *Some Aspects of the Ancient Civilization of South India.*

The Michigan Section of the Mathematical Association of America held its fall meeting at Albion College, Albion, Michigan, Saturday, November 28, 1936.

1. *A simplification of the calendar.* Professor W. D. Baten, Michigan State College.

2. Papers presented by the Committee on Undergraduate Interests.
   a. *A locus problem.* Mr. Richard Fowler, '36, Albion College.
   b. *Complex numbers and triangles.* Mr. Paul Nims, '37, University of Michigan.
   c. *A study of the cardioid by inversion.* Miss Violet Davis, '36, University of Toledo.

3. *The generalization of the theorem of Pythagoras to three-dimensional space.* Mr. D. K. Kazarinoff, University of Michigan. Introduced by the Secretary.

4. *A generalization of a theorem concerning harmonic functions.* Dr. Max Coral, Wayne University. Introduced by Professor A. L. Nelson.

5. *Some results of a testing program at Michigan State College.* Dean L. C. Emmons, Michigan State College.

6. *Conics and their inverses.* Professor L. S. Johnston, University of Detroit.

7. *Note on the class number function.* Dr. J. D. Elder, University of Michigan. Introduced by the Secretary.

8. *J. L. Lagrange, on the 200th anniversary of his birth.* Professor G. Y. Rainich, University of Michigan.

Dr. Vijayaraghavan, one of India's most promising young scholars, is this year's visiting lecturer of the American Mathematical Society. He was a student of G. H. Hardy at Oxford, and has done important research in connection with Tauberian theorems, continued fractions and differential equations. Dr. Vijayaraghavan was born in the same village as the famous Indian mathematician Ramanujan.

Mathematics Section meetings were held October 28-30, 1936, as a part of the Nebraska State Teachers' Association in each of the six districts as follows:

District No. 1 at Lincoln, Friday afternoon, October 29th

President, Rinaldo Bacon, Lincoln.
Secretary, Margaret Morton, Fairbury.
Program:

A Film.

Address by Dean O. J. Ferguson, University of Nebraska.

Tour of the Mechanical Engineering Laboratories.

District No. 2 at Omaha, Friday afternoon, October 29th

President, Amanda E. Anderson, Central high school, Omaha.
Secretary, Paul Morris, Kennard.
Program:

The Use of Objective Tests for Improving Instruction in Secondary Mathematics by Dr. H. E. Schrammel, Teachers' College, Emporia, Kansas.

Round Table Discussions: *Cultural Aims in the Study of Mathematics.*

Art Objectives, Maud Searson, Benson High School, Omaha.

Formation of Correct Habits and Attitudes, J. F. Woolery, formerly Central High School, Omaha.

Appreciation of the Contributions of Mathematics to Civilization, by Walter J. Luebke, S. J., Creighton University high school, Omaha.

District No. 3 at Norfolk, Thursday afternoon, October 27th

Chairman, Amy Yorke, Norfolk.

My Method of Dealing With the Problem of Inaccuracy, Lena Porter, Elgin.

The Method of Caring for the Difficulties in Problem Solving, Floyd Schelby, Madison.

Motivation in Mathematics, Miss Enid Conklyn, Wayne.

Mathematics for Use, Dr. H. E. Schrammel, Kansas State Teachers' College, Emporia, Kansas.

District No. 4 at Kearney, Friday afternoon, October 29th

President, Inez Burnworth, Lincoln.
Secretary, O. L. Splinter, Calloway.
Program:

Modern Trends in Secondary Mathematics, Dr. R. W. Deal, Nebraska Wesleyan University.

Report of the Delegate Assembly, Miss Eva Phalen, Kearney.

Graduate Study for the High School Mathematics Teacher, Clarence H. Lindahl, Paxton.

District No. 5 at McCook, Friday afternoon, October 29th
Science and Mathematics Section

Chairman, Herbert Finke, Holdrege.
Program:

Teaching Pupils to Study Science, Dr. W. L. Beauchamp, University of Chicago.

District No. 6 at Alliance, Friday afternoon, October 29th

President, Carl Thomas, Chadron Teachers College.
Program:

How-to-study Program for Mathematics, Supt. F. H. Holmgrain, Merriman.

Should Mathematics be a Required Subject in High School? Kenneth Rawson, Kimball.

What is the Relationship of Mathematics to the Social Sciences? Dr. A. R. Congdon, University of Nebraska.

The above summary was collected and sent in by Prof. Arthur L. Hill, Peru State Teachers' College, Peru, Nebraska.

Book Reviews

Edited by
P. K. SMITH

Johannes Kepler als Mathematiker. By Fritz Kubach. (Vol. II in the series "*Veröffentlichungen der Badischen Sternwarte zu Heidelberg, Königstuhl*," edited by Heinrich Vogt), Karlsruhe, 1935; 83 pp., in paper covers.

Kubach has written this monograph on Kepler's mathematical activity with special reference to the unity of his life-work. He feels that Kepler's mathematical creations are only one phase of the forces and decisions in his life, to which all his other activity is subordinated. The biographical introduction is based on printed sources and letters. The Kepler bibliography is an attempt to list all German and Latin editions of Kepler's works and correspondence, and to include German works about Kepler and essential historical material about him in other publications. From this viewpoint, it will have a certain value; only material in the German language is listed in this bibliography.

The author was able to find almost nothing concerning the mathematical instruction which Kepler received. Probably any such material was destroyed in the Thirty Years' War. In a letter to Herwart, dated March 26, 1598, Kepler gives 1594 as the year in which he began really to busy himself with mathematics. At the beginning of that year he was called to Graz as professor of mathematics.

Chapters III and IV then take up in considerable detail Kepler's work as professor of mathematics in Graz and Linz; in both of them original documents are freely quoted and no doubt the Kepler specialist will find much of interest. There follows a well organized chapter on Kepler's mathematical accomplishments in general. Kubach then gives pertinent excerpts from contemporary documents dealing with the Graz and Linz periods, which, coupled with the discussion of these periods, is of course the principal contribution of this volume. The bibliography shows how much Kepler literature there is, but this monograph should be in the library of anyone interested in Kepler, as well as others.

University of Illinois. G. WALDO DUNNINGTON.

Introduction to Mathematics of Business. By W. L. Hart. D. C. Heath and Company, New York, 1936. VIII+321 pages exclusive of 100 pages of answers and tables.

This text is intended as a basis of a course of three hours per week for one session for students specializing in applications of mathematics to business, just as its name implies. It contains nineteen chapters: ten chapters devoted to topics taught in algebra generally, one chapter given to statistics, five chapters treating annuities, and three chapters covering annuities as applied to insurance.

At least one year of elementary algebra is expected as a prerequisite; and, in particular, a certain mathematical maturity is needed. Excepting chapter IX, the first eleven chapters dealing with ordinary algebra treat, in order, the following topics: The fundamental operations of algebra, fractions, radicals and exponents, equations, significant digits and related topics, simple interest and simple discount, functions and graphs, systems of linear equations, logarithms, compound interest, and progressions. The author has endeavored to present these topics in the smallest compass commensurate with wisdom.

Chapter IX is devoted to topics related to statistics. Treatment is made of the arithmetic mean, the median, index numbers, the straight line of best fit, and time series. Suitable exercises are included. Nothing is said of the geometric and harmonic means and the mode. However, the author himself frankly states, "This chapter does not claim to be a rounded introduction to the field of statistics as it appears in its collegiate applications."

The remaining chapters of the book, treating life annuities and life insurance, omit the more complicated notations peculiar to these topics and make their chief aim the concept of the present value of a contingent payment, the computation of the net single premium and net annual premiums, and a clear notion of a policy reserve.

The author has marked with a star certain chapters and paragraphs which may be omitted for courses shorter than the regular course of 3 hours per week, for one session, designated as Course 1. By observance of the suggestions made, the instructor may choose between Course 2, 5 hours per week for one semester, and Course 3, 3 hours per week for one semester.

Answers to the odd-numbered exercises are given in the book, but answers to the even numbered ones may be obtained in a separate pamphlet. Suitable tables also are given as a part of the text.

As a piece of editorial work, the book is carefully written, the material well arranged on the page, and the binding and color attractive.

Louisiana State University. IRBY C. NICHOLS.

Plane Trigonometry. By K. B. Patterson and A. O. Hickson, Duke University. F. S. Crofts and Co., New York, 1936. ix+220 pages.

Since trigonometry is fundamental to nearly all the sciences and extensively usable in them the authors encourage the student to use trigonometry in learning it and to create exercises for himself. They give suggestions for making original identities and practical problems involving triangles to be solved. At the ends of the chapters they list exercises that have originated in their classes and provide blank pages for the student to record exercises he has made and solved.

Each chapter has a definite objective in subject matter that the student should learn by study, by working assigned exercises, and by making and doing original exercises. Although major attention is devoted to analytical trigonometry, the solution of triangles is fully treated. There is a good chapter on logarithms. Full explanation is made as to the uses of the various tables. The textbook is abundantly supplied with exercises. Answers to the authors' exercises are listed in the back of the book, while the answers to the students' original exercises are given with the exercises.

The authors say in the preface that to follow their plan requires more time on the part of the instructor but that the time lost to the instructor is gained many times over by the student in aroused interest, understanding, and confidence. The authors and their colleagues have used the textbook in mimeograph form for a number of years. The textbook is bound with or without the revised (1935) edition of E. S. Crawley's Tables of Logarithms to Five Places of Decimals with Auxiliary Tables.

Vanderbilt University. WILSON L. MISER.

Elementary College Algebra. By H. W. Kuhn and J. H. Weaver. Macmillan Company, New York, 1935. xxvi+359 pages.

In the preface the authors set forth their purpose to meet three needs of the student: (1) sufficient review material on high school mathematics, (2) arrangement of material so as to meet the needs of various college groups, (3) presentation of material in a simple and logical form within the reach of the maturity of the student.

The first five chapters cover review material with ample exercises and problems. Chapter VI is on systems of linear equations. This chapter concludes with a discussion of indeterminate equations. Exponents, radicals, and quadratic equations in one unknown are treated in the next two chapters. Immediately following the chapter on quadratics in one unknown come two chapters on inequalities, ratio, proportion, and variation. These topics usually precede the next chapter, which in most college algebra texts is on simultaneous quadratics. Logarithms and progressions are treated next. The chapter on logarithms includes a section on semilogarithmic and logarithmic coordinate paper. In chapter XIV fifteen pages are devoted to the mathematics of finance. The binomial theorem, complex numbers, integral rational functions, and theory of equations are the topics in the following four chapters. In the chapter on integral rational functions the concept of the derivative is taken up. The method of finding the derivative of a polynomial is given, as well as a few simple problems on maxima and minima. The introduction to the calculus is carefully done. In the chapter on the theory of equations the method of interpolation, Newton's method, and Horner's method are given for solving numerical algebraic and transcendental equations. The text ends with chapters on permutations and combinations, probability, determinants, and partial fractions.

The authors attain their three main objectives with reasonable satisfaction. The arrangement of the material is good throughout. The development of the theory is critical and at the same time not too difficult for the student to follow.

The text is subject to criticism with reference to errors in the answers to numerous problems. A few of these errors occur in the following: Ex. 15, p. 33; ex. 32, p. 34; prob. 6, p. 41; prob. 19, p. 42 (poorly expressed); prob. 24, p. 50; prob. 26, p. 50; ex. 39, p. 73; ex. 34, p. 85; prob. 69, p. 95, and prob. 19, p. 108. In the solution of ex. 34, p. 85 after $i\sqrt{3}$ has been transposed and each member squared the value $-2-2i\sqrt{3}$ is gotten for x. This value should then be substituted to test for equivalence. This testing would require the extracting of the square root of a complex number. At this place in the text no such process has been taken up. So the exercise seems to be out of place.

Heavy type has been used for emphasizing important terms. The format is excellent. This text should be examined by anyone seeking a splendid college algebra.

Louisiana Polytechnic Institute. P. K. SMITH.